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Hooke's Chain Theory and the Construction of Catenary Arches in Spain

Josep Lluis I Ginovart [a], Sergio Coll-Pla [b], Agustí Costa-Jover [b], and Mónica López Piquer [a]

[a]School of Architecture UIC Barcelona, Universitat Internacional de Catalunya, Barcelona, Spain; [b]Escuela Tècnica Superior de Arquitectura Reus, Universidad Rovira i Virgili, Reus, Spain

ABSTRACT

Catenary arches were used in Spanish Art Nouveau architecture by Antonio Gaudí (1852–1926). The theory of the chain, which is based in the shape of a hanging collar, was proposed by Robert Hooke (1676) and was used by Christopher Wren in Saint Paul's dome (1675). British school modern mechanics theory was introduced in Spain by Spanish borbonic military engineers as well as by Catholic Scottish and Irish families during the eighteenth century.

The assessment of some drawings of gunpowder warehouses, which were found in the collection of Mapas planos y Dibujos (MPD) of the General Archive of Simancas (Archivo General de Simancas, AGS) (AGS 2014), have revealed the use of the chain theory in Miguel Marín's projects for Tortosa (1731) and Barcelona (1731) as well as in Juan de la Feriére projects in A Coruña (1736). Built evidence has also been found: The Carlón wine cellars in Benicarló, which were built by the Irish O'Connor family (1757). The analysis of these examples demonstrated the arrival the chain theory to Spain during the first half of the 18th century.

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catenary; geometry; gunpowder warehouse; military engineering; Robert Hooke

1. Introduction

Catenary arches are one of the main features of Art Nouveau architecture in Spain. Their shape is based on the modern theory of masonry arches. This theory was developed during the nineteenth century, and the work of Antonio Gaudí (1852–1926) is claimed as its main exponent (Huerta 2004). However, 50 years before Antonio Gaudí, Catholic families that emigrated from Scotland and Ireland had already initiated some of the catenary's form of mathematical theory in some practical uses; this was the theory that began to be taught at the Mathematics Academy of Barcelona in 1720.

This article addresses the introduction of the concept of the catenary arch in Spain before the 19th century. After an exhaustive review of the theoretical framework, some cases are assessed. The aim of the research is to determine whether the mechanical concept of the chain was used by Spanish military engineers and by exiled English engineers, who built several wine cellars in Spain. Thus, we intend to determine whether there is any geometrical relationship between the layout of several gunpowder magazines that were made by Spanish military engineers in the 1730s and the construction of a civil building—the Carlón wine cellar in Benicarló (1757)—in which catenary arches may have been used.

A historical approach to British and Catholic military engineers was presented at The Second Construction History Society Conference under the title *Hooke's chain in the Spanish Enlightenment of the XVIIIth century* (Lluis I Ginovart 2015a). It was a geometrical assessment of the Benicarló's cellar arches, which were scanned with a Total Station by J. Lluis, A. Costa, and S. Coll (2014). Due to the technical limitations of the methodology, a further survey was conducted specifically for the study that is presented in this paper using Terrestrial Laser Scanner techniques, J. Lluis, A. Costa, and S. Coll (2014–15). That made it possible to improve the initial survey. The accuracy of the newly obtained data, together with the statistical comparison of the arches' shape with other geometrical forms, completes the initial study on the 18th-century introduction of catenary arches in Spain.

2. Theoretical framework

2.1. The hanging chain theory and its application to arch construction

The theory of the chain was proposed by Robert Hooke (1635–1703) at the end of his treatise, *A description of helioscopes, and some other instruments* (1676). The

CONTACT Josep Lluis I Ginovart recercapatrimoni@urv.cat C/Universitat 1, Campus Bellissens, ETSA, 43204 Reus, Spain.

awareness of the shape of the catenary was applied by Christopher Wren (1632–1723) in Saint Paul's dome (1675), which was designed in collaboration with Robert Hooke (Heyman 2003).

Years later, David Gregory (1659–1708) formulated the equation of the catenary and affirmed that the catenary was the real shape of an arch: if this type of arch can sustain itself, then it is because a catenary can be drawn in its section (Gregory 1676). In *Lineae Tertii Ordinis Neutonianae* (1717), James Stirling (1692–1770) compiled some ideas that were advanced by the English School of Mathematics and described how to build a catenary with hanging spheres to simulate the workings of a constructive element (Stirling 1717). This solution inspired Giovanni Poleni (1683–1761) in *Memorie istoriche della Gran Cupola del Tempio Vaticano* (1748) (Poleni 1748) to develop a similar methodology to Stirlings' to explain the collapse of the vault of San Pietro's Basilica (Heyman 1988). In the same way, Pierre Couplet (1670–1744), in his work *De la poussée des voûtes*, also refers to the chaînette (the hanging chain) as the best of all possible shapes for vault construction (Couplet 1729).

Furthermore, in *Traité de méchanique* (1695), Philippe de La Hire formulated the principles of the equilibrium of arches and vaults under the theory of the quoin. Bélidor at *Nouveau cours de Mathématique à l'usage de l'Artillerie et du Génie, où l'on applique les parties les plus útiles de cette science à la théorie et à la pratique de différents sujets qui peuvent avoir rapport à la guerre* (1725) simplified de La Hire's method to a data table, essentially reducing it to the required abutment for construction. In *Essai sur une application de maximis et minimis a quelques problemes de statique, relatifs a l'Architecture* (1773), the engineer Charles-Augustin Coulomb (1736–1806) determined the existence of maximum and minimum strength in an arch.

The introduction of these theories in Spain occurred in the context of the Mathematics' Academy of Barcelona (1720), where the main reference was the work of Bernard Forest of Bélidor (1698–1761), *La science des ingénieurs, dans la conduite des travaux de fortification et d'architeture civile* (Bélidor 1729), which was partially translated by John Müller (1699–1784). It enables nineteenth-century authors such as Claude-Louis-Marie-Henri Navier (1785–1836) in *Resumé des Leçons donnés à l'Ecole des Ponts et Chaussés sur l'Application de la Mécanique à l'Etablissement des Constructions et des Machines* (Navier 1826), E. Méry in le *Mémoire sur l'equilibre des voûtes en berceau* (Méry 1840), and Henry Moseley (1802–1872) in the *Mechanical Principles of Engineering and Architecture* (Moseley 1843) to complete the elastic theory of masonry arches and vaults. A more comprehensive revision of the theoretical framework about the catenary theory can be found in (Lluis I Ginovart 2015b).

These theories were also the basis of the modern theory for the analysis of masonry structures, as expressed by Jackes Heyman, who resumed the development of the theories between the 18th and 19th centuries and stated the principles of limit analysis to assess the safety of masonry arches (Heyman 1966, 1995). Other authors such as O'Dwyer (O'Dwyer 1999), Huerta (Huerta 2005), and Block et al. (Block, Ciblac, and Ochsendorf 2006), have developed these principles. Further investigations have focused on specific problems such as the minimum thickness of masonry structures (Alexakis and Makris 2013) (Coccia, Como, and Di Carlo 2016) and they have also determined the limit equilibrium states of historical structures.

2.2. The libraries of the military engineers of the 18th century

The principal function of the Mathematics Academy of Barcelona (1720) was the compilation of scientific works for its library. The bibliographic interest of engineers, who had their own libraries, led Vicente Garcia de la Huerta (1734–1787) to publish the Biblioteca Militar Española (1760) with the catalogue of the main treatises of the sixteenth and seventeenth centuries (García De La Huerta 1760).

Some of the most relevant texts that were available for military engineers in the libraries are: *L'architecture des voûtes* (1643) by François Derand (1588–1644), in the library of Jorge Prosper Verboom (1665–1744), and the *Treatise on Stereotomy* by Abraham Bosse (1604–1676), in addition to *La pratique du trait à preuve de M. des Argues Lyonnois pour la coupe des pierres en Architecture practiquer la perspective* (1643), in the Library of the Barcelona Academy.

However, the main references for the Spanish military engineers were definitely the works of Bernard Forest of Bélidor (1698–1761): the *Nouveau cours de Mathématique* (1725), *La science des ingénieurs* (1729), and the two volumes of his *Architecture hydraulique* (1737–1739). (*Library of the Academia, Verboom (+1744), Espinosa (+1787), Burgo (+1788), Cermeño (+1790) Y Miguel de Roncali (+1794)*).

2.3. Gunpowder warehouses in the military architecture treatises

Military architecture treatises that were developed at the end of the seventeenth and eighteenth centuries

make reference to the construction of these warehouses, especially if they have an element of high resistance, such as having been made bombproof or having been constructed underground (whether in manmade excavations or in caves) (Cassani 1705). The principal work of reference is *Maniere de fortifier selon la methode de Monsieur de Vauban*, of Sébastien Le Prestre, Vauban (1633–1707), which was edited by the abbot Du Fay in 1681. The morphology of the gunpowder warehouses with a double enclosure is defined in that treatise, together with the design of its roofing (Fay 1687). In the second edition of the work, in 1693, a double enclosure with a central body and canon vault, on walls with stirrups, is drawn.

In the Spanish treatise *El Ingeniero Primera Parte, de la Moderna Architectura Militar (1687), by Sebastián Fernández de Medrano* (1646–1705), that question is solved with a masonry vault of four feet (0.324 m) width of Spanish "vara". Subsequently, in the *El Architecto Perfecto en el Arte Militar* (1700), the thickness is increased from 12–14 ft (Fernández 1700). In the *Escuela de Palas ò sea Curso Mathematico* (1693), attributed to José Chafrion (1653–1698) in the *Libro de Arte Militar*, XI Treatise (Chafrion 1693), there are constructions that are similar to the Vauban's constructions. This vault is solid, and it is built with lime masonry, which differs from Sebastián Fernández's vault.

In *Le Nouveau cours de Mathématique* (1725), Bernard Forest of Bélidor, considered a practical application of masonry mechanics for the construction of gunpowder warehouses (Belidor 1725). He determined the abutment for a canon vault and for a tiers-point pointed arch. In a table, he synthesized the dimension of the pieds droits, in ratio with its curve and localization. The importance of these strategic elements obligates Bélidor to dedicate the entirety of Chapter No. 9 of the IV Book of *La science des ingénieurs dans la conduite des travaux de fortification et architecture civile* (Bélidor, 1729) to the construction of gunpowder warehouses. In *La science des ingénieurs* (Bélidor, 1729) (Galindo Díaz 2000), IInd Book, IIIrd Chapt., 5th Proposition, Bélidor presented the curve that must be given to a vault not only to maintain its weight throughout but also to maintain its equilibrium. As a result, its curve would take the shape of a catenary. Thus, for military constructions, Bélidor determined up to five different types of vaults: semicircular, third-tip pointed, elliptical (drawn as a segmental arch), plane and (the derived forms of the) catenary.

The work of Bélidor was translated into English by John Müller (1699–1784) and published under the title A treatise containing the elementary part of fortification, regular and irregular. For the use of the Royal academy of artillery at Woolwich (1755). This text was also translated into Spanish by the Mathematics Academy professor Miguel Sánchez Taramas in Barcelona (1769) (Pita 2009) under the title *Tratado de fortificación, ó Arte de construir los edificios militares, y civiles* for the use of the pupils of the Mathematics School. For the construction of gunpowder warehouses, their references are Vauban (1681) and Bélidor (1729) texts (Müller 1769).

2.3.1. Civil constructions of the 18th century

After the Bourbon dynasty's ascendancy to the Spanish throne (in the year 1700), Catholic diplomatic and military families of Irish and Scottish origin emigrated under royal protection, which preserved their status. The O'Connor family was installed in Benicarló in the 18th century, and they were associated with the McDonnells in the wine export business.

The Bourbon dynasty, which established itself in Spain in 1700 with King Philip V (1683–1746), created the Army Corps of Engineers by the Royal Decree of April 17, 1711. Several Irish families moved to eastern Spain in the mid-17th century. Patrick White Limerich, who was a trader in agricultural products and wine, and the O'Connor family settled in Benicarló in 1749. Gaspar White and the O'Gorman family were based in Alicante, whereas the Lilikells, the Tuppers and Henry O'Shea lived in Valencia. Against this backdrop, the O'Connor family built a Carlón wine cellar in Benicarló in 1757 (Constante Lluch 2012), which is the subject structure of this article.

Initially, the Bourbon engineers, depending on the origin of their families, used either the vara of Burgos, which was enforced by Philip II on June 24, 1568, or Philip V's toise. The discrepancies among some of the Spanish military engineers (especially between the Academy's educational activity, in which the vara was used, and common practice, in which the toise was used) made it necessary to issue the memorandum of July 14, 1750 for the Captain Generals, which not only established the use of the Castilian vara (instead of the toise) for the teaching of Mathematics but also in all activities that pertained to the army and the navy. King Ferdinand VII reversed this circular note through the Royal Order of February 14, 1751, which stated that the toise had to be used in all military construction.

The wine cellar's construction, which uses diaphragm arches, is very similar to the gunpowder magazine projects that were built by the Army Corps of Engineers in a neighbouring geographical area. These include the project by Carlos Beranger [MPD, 06, 169] for Benimamet (1751) and the project by Juan Bautista French (1756) for Peñíscola [MPD, 07, 208]. In these projects, as opposed to the O'Connor cellar, the arch

abutment is on the outside of the building. Nevertheless, there is a subsequent project by Antonio López Sopeña [MPD, 28, 027] for A. Coruña (1774), in which he uses diaphragm arches with inside abutments that are similar to those that were used in the Benicarló's wine cellar.

3. Method

The formal relationship between the projects of several gunpowder magazines and the Benicarló's wine cellar (built in 1757) is analysed from proportional and geometrical perspectives.

A total of 74 gunpowder warehouse projects, that are preserved at the General Archive of Simancas (which exists thanks to the teachings of the (1715–1798) Mathematical Military Academy of Barcelona), have been studied (AGS 2014). Three studies have been found among them, in which the mechanical concept of the catenary could have been used. The first two were drawn by Miguel Marín, who developed gunpowder warehouses in Barcelona (1731) and Tortosa (1733). The third, which is a warehouse in A Coruña (1736) (Figure 1), was projected by Juan de la Feriére.

For Benicarló's arches analysis, we have used the obtained data in the topographical survey by J. Lluis, A. Costa, and S. Coll (2014–15). The device used is a Leica ScanStation C10. The field of view is in horizontal 360º and in vertical it is 270º, and the integrated colour digital camera has a resolution of 1920x1920 pixels (4 megapixels). Next, the obtained point clouds were used to draw a section line for each of the eight arches that close off the space of the cellar (Figure 2). The geometrical analysis of O'Connor's cellar arches have taken as a reference the proportions that were identified in the geometrical assessment of the three studies of the gunpowder warehouses.

The geometry of the arches is compared with the geometric constructions of an oval (O), an ellipse (E) and a catenary (C) because they are the geometric forms that were proposed by Bélidor at *La science des ingénieurs, dans la conduite des travaux de fortification et d'architeture civile* (1729). This allows a statistical analysis of metrical deviations. The geometrical definition of these forms is done based on the following measure criteria.

- Ovals (O): having the same span (e1) and the same rise (e2) as the arch to be compared. The rise is determined on the basis of the maximum arch height from the springline, plus the relative eccentricity with respect to the springline, which is considered to be one foot (just as in the projects for the gunpowder magazines).

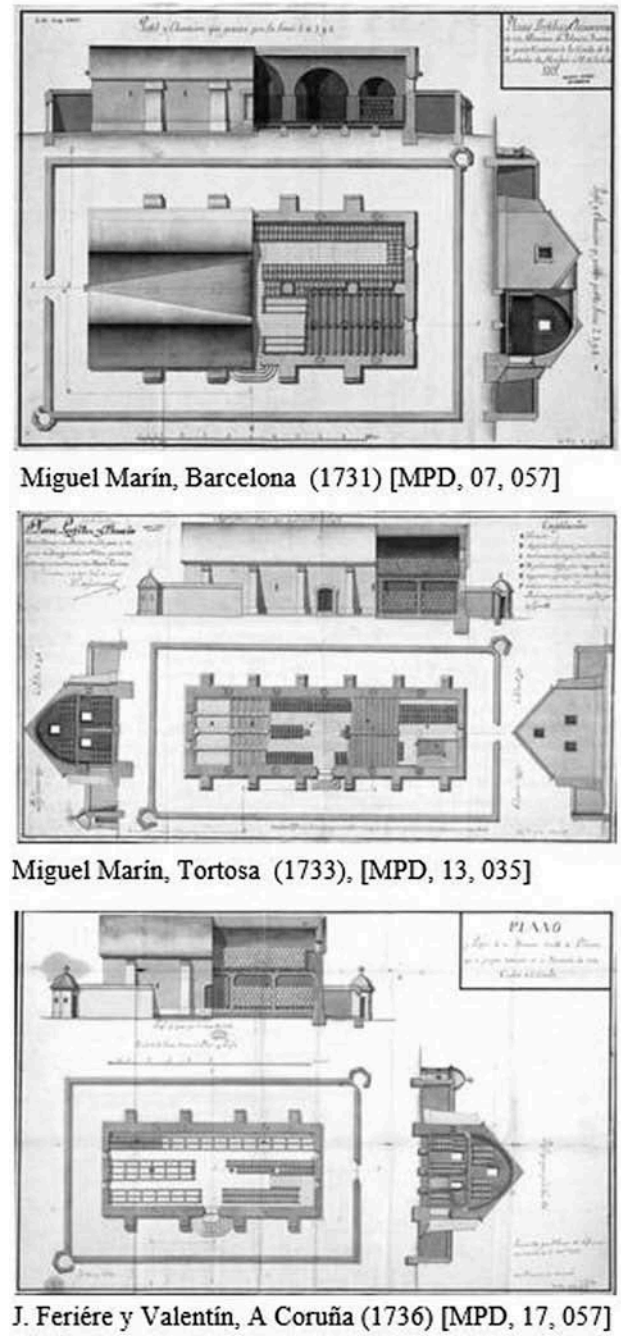


Figure 1. Gunpowder warehouses: 1.1: M. Marín (1731); 1.2: M. Marín (1733); 1.3: J. Feriére y Valentín (1736).

- Ellipses (E): having the same dimensions (e1, e2) as the ovals.
- Catenaries (C): having the same span (e1) and the same rise (e2) as the arch to be compared and the maximum arch height from the springline.

The comparison was performed by drawing the reference curves (the Oval, the Ellipse, and the Catenary) with a CAD application (Computer Aided

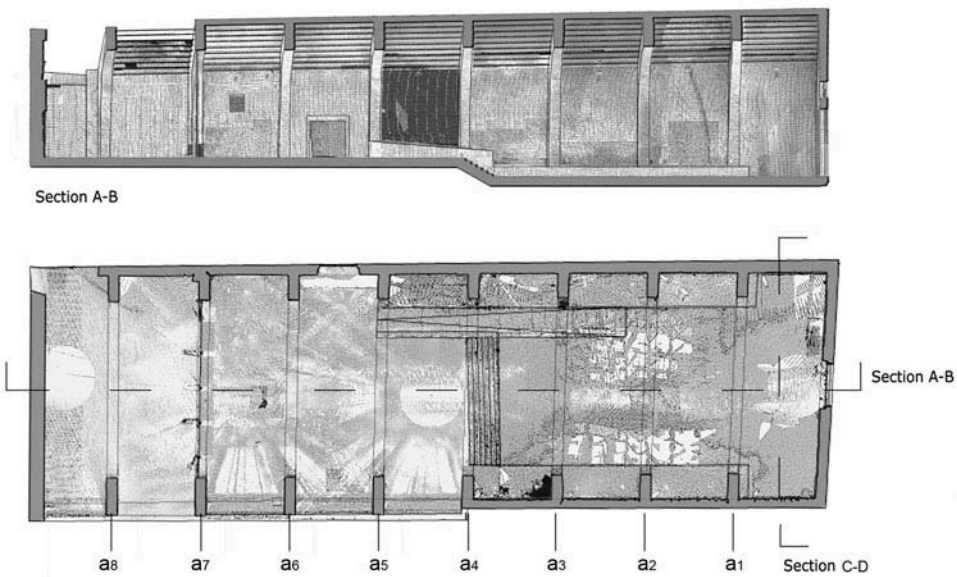


Figure 2. Floor plan and cross section of the O'Connor cellar in Benicarló (1757).

Design), and comparing it with the curves from the gunpowder magazine projects and the ones that were obtained from the survey that was performed by J. Lluis, A. Costa, and S. Coll (2014–15) in the Benicarló wine cellar.

Three common points are fixed: span (e1), two fascias ($-x_1$, 0) and (x_1, 0), rise (e2), vertex (0, y_1), and (x_2, 0), (0, y_2) as variable coordinates of the ovals' centers. Thus, the surveyed curves $f(x_a)$ are compared with the functions of c atenary [1], semi-oval [2], and semi-ellipse [3]:

$$f(x_c) = a \cdot Cosh\left(\frac{x}{a}\right) \quad [1]$$

$$f(x_0) = \frac{2y_2^2(x_1 - x_2)}{y_2^2 + (x_1 - (x_1 - x_2))^2} \quad [2]$$

$$f(x_e) = \frac{y1}{2x_1\sqrt[2]{1,5x_1}}. \quad [3]$$

In [1], $a = (T_o/P)$; T_o is the horizontal component of the tension, which is constant, and P is the weight per length unit.

The length of the assessed arches $Lf(x_a)$, is compared with the catenary [4], the semi-oval [5], and semi-ellipse [6]:

$$Lf(x_c) = \int_{x_1}^{-x_1} a \cdot Cosh\left(\frac{x}{a}\right) .dx \quad [4]$$

$$Lf(x_0) = \int_{x_1}^{-x_1} \frac{2y_2^2(x_1 - x_2)}{y_2^2 + (x_1 - (x_1 - x_2))^2} \cdot dx \quad [5]$$

$$Lf(x_e) = \int_{x_1}^{-x_1} \frac{y1}{2x_1\sqrt[2]{1,5x_1}} \cdot dx \quad [6]$$

Also, the area that is defined by the arches $Sf(x_a)$ is compared with the areas of the catenary [7], the semi-oval [8], and the semi-ellipse [9]:

$$Sf(x_c) = \int_{x1}^{-x1}\int_{0}^{y1} a \cdot Cosh\left(\frac{x}{a}\right) \cdot dxdy \quad [7]$$

$$Sf(x_0) = \int_{x1}^{-x1}\int_{0}^{y1} \frac{2y_2^2(x_1 - x_2)}{y_2^2 + (x_1 - (x_1 - x_2))^2} \cdot dxdy \quad [8]$$

$$Sf(x_e) = \int_{x1}^{-x1}\int_{0}^{y1} \frac{y1}{2x_1\sqrt[2]{1,5x_1}} \cdot dxdy \quad [9]$$

The process is simplified by the computer application Innersoft, a plugin for AutoCAD that installs a suite of productivity tools. That enables to vector path curves through polylines, which sets an error of ($1x10^{-11}$) and a number of divisions depending on the length of the curve.

The ovals that pass through the three points are laid out by means of Tosca's method (Tosca 1712), for which the center is placed on the major axis (y). With regard to the cellar arches, the center is also placed on the major axis (y), taking into account the eccentricity of the springline. The centers on the (x) axis are found using the Tosca method; next, we obtain an oval (Oy). The center of each oval is placed on the (y) axis, according to the ratio between the length of the semi-major axis and the radius of the minor arc (the same ratio that was found in the gunpowder magazines). Using the ratios that were obtained in the proportional assessment for the gunpowder magazines projects, it is possible to draw the ovals [O1, O2, O3]. Although these

ovals are not the ones that most closely resemble arches $a_{(1-8)}$, they do establish an interval of possible results, which is necessary because an infinite number of Tosca ovals could theoretically pass through the three end-points of these arches.

4. Proportional study in military construction

In the construction of gunpowder warehouses, barrel and pointed vaults are generally used, although there are some examples with elliptical vaults, such as that one that was built in 1694 by Hércules Torelli in Pamplona. This construction was remodelled by Francisco Larrando de Mauleon in 1718 [MPD, 31,031] (Laorden 2005). Mauleón was a professor at the Mathematics School of Barcelona and Zaragoza and authored the *Estoque de la Guerra y Arte Militar*, which was published in Barcelona (1699). The Viceroy had ordered the repair of the fortifications and gunpowder warehouses to make them bombproof. The elliptical vault was replaced by a barrel vault to make it less visible and vulnerable to enemy artillery. It was reinforced and reduced in height according to the concept that was introduced in the military treatises by General Ambroise (d. 1587) in *Le Timon du Capitaine* (1587) (Bachot 1587).

The simple vault of the warehouse of Montjuic mountain in Barcelona (1731) [MPD, 07, 057], which is a project that is attributed to Miguel Marín, is not generated through an arch of circumference. The geometric study reveals that the vault has a span of 16 feet in toesa, a rise of 11.5 ft, a wall with a width of 3 ft and a buttress of 7 ft. Other projects include the project from Miguel Marín for Tortosa (1733) [MPD, 13, 035], which has a span of 21 ft in toesa, a rise of 14 ft, a wall with a width of 3.5 ft and a buttress of 7 ft. Another similar project is the simple warehouse layout by Juan de la Feriére y Valentín in A Coruña (1736) [MPD, 17, 057], which has a span of 22 ft in toesa, a rise of 14 ft, a wall with a width of 3 ft and a buttress of 7 ft. (Figure 3)

The design of the pointed vault is initially compared with the catenary, as it was obtained with a chain over a reproduction of the plans on a larger scale (Figure 4). Thus, the arch that is described by the chain is very similar, but it is not coincidental to the profile of the vault because there are small deviations near the springline of the vault. This deviation is due to the fact that it is not possible to lay out the catenary with traditional drawing tools, such as rulers and compasses.

The assessment of the original drawing of the section of the warehouse reveals three compass marks. One point is made over the vertical axis of symmetry of the figure, while the other two are made over the perpendicular axis, which is positioned slightly below the springline of the vault. An oval was drawn on each project using these compass marks, and the obtained curves were coincident with the curves of the projects. Thus, to draw the projects of the warehouses, both Miguel Marín and Juan de Feriére y Valentín used the geometrical solution of an oval.

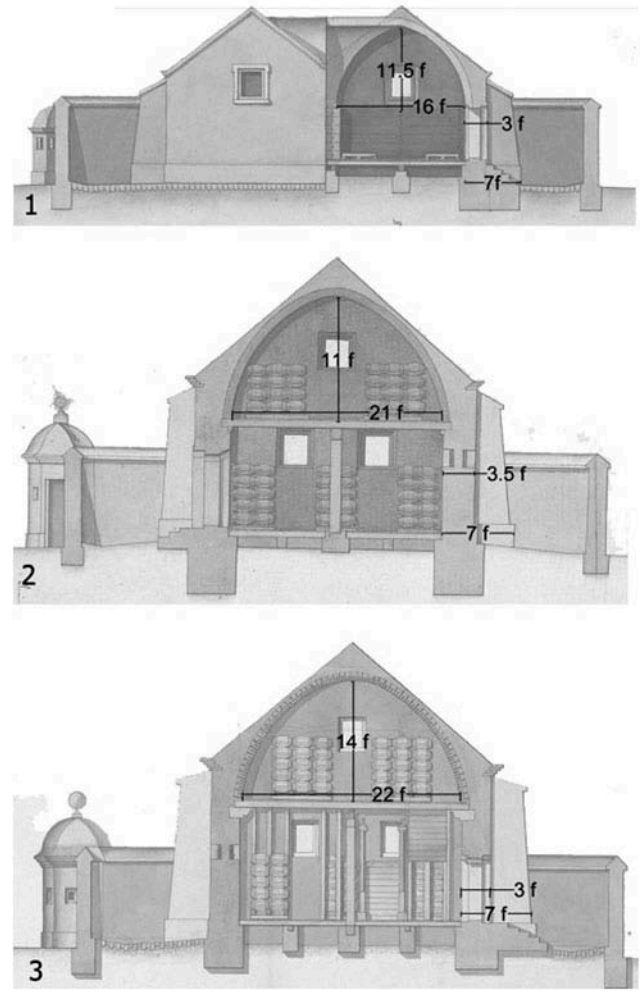
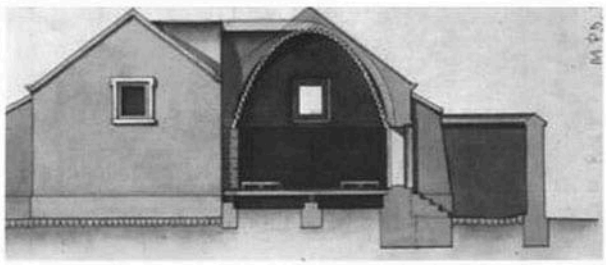


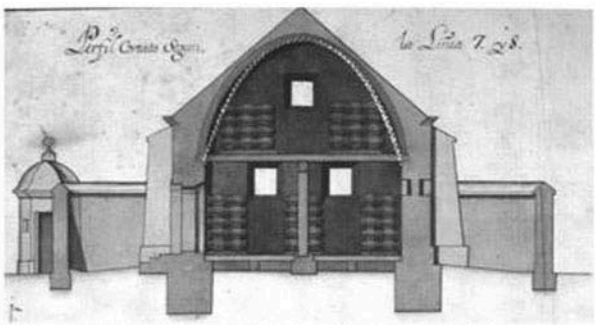
Figure 3. Metrology of the projects for the gunpowder magazines.

Therefore, the curves that are drawn in the three projects are ovals; however, the major axis of the oval is higher than the springline of the arch. As a consequence, the curves are not tangential at the springing. Thus, the military engineers drew the curve of the vault as an arch *apaynelado, carpanel* of Tosca (1712), or *anse de panier* of Bélidor (1729). (Figure 5).

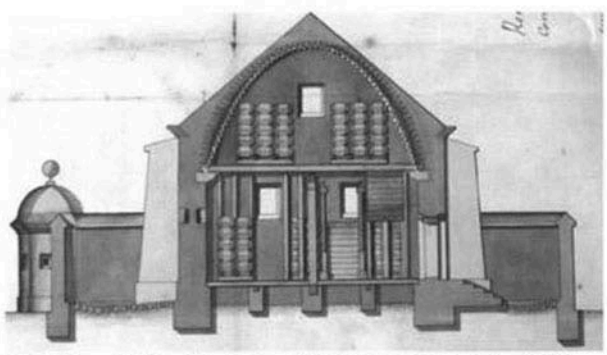
The fact that the geometric layout of these vaults, which is based on ovals, was well known by eighteenth-century military engineers. They began from the essential feature that oval vaults are tangential to the springline of these building elements. When the springline is higher than the axis, a non-tangential curve is obtained, which is



Miguel Marín, Barcelona (1731) [MPD, 07, 057]

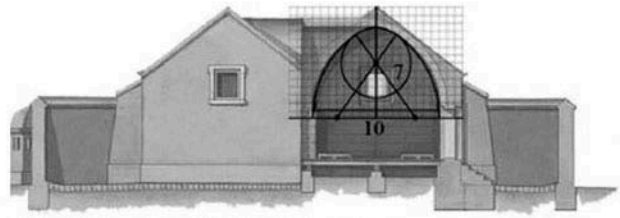


Miguel Marín, Tortosa (1733), [MPD, 13, 035]

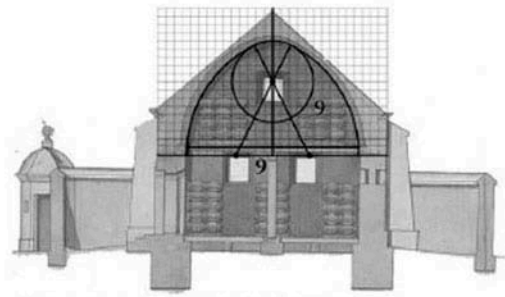


J. Feriére y Valentín, A Coruña (1736) [MPD, 17, 057]

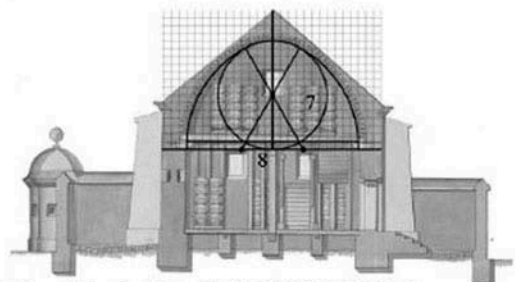
Figure 4. Chaînette method applied to the projects for the gunpowder magazines.



Miguel Marín, Barcelona (1731) [MPD, 07, 057]



Miguel Marín, Tortosa (1733), [MPD, 13, 035]



J. Feriére y Valentín, A Coruña (1736) [MPD, 17, 057]

Figure 5. Oval method applied to the projects for the gunpowder magazines.

a feature of the catenary definitions that are given by Frézier (1738). Concurrently, Bélidor (1729) specifies the method to lay out the true shape of the catenary vault. By knowing the rise and the span of the vault, the architectonic shape is determined with a hanging chain. Thus, a scale model can be built and can easily be taken to the construction site. By contrast, the layout of the catenary in military engineers' projects is more complex because it requires the use of an approximation of the catenary through the geometrical shape of a lowered oval.

The ovals are derived from the centers of the circumferences (the compass center points on the paper). They are referred to: (O1) for Barcelona (1731) [MPD, 07, 057], (O2) for Tortosa (1733) [MPD, 13, 035], and (O3) for A Coruña (1736) [MPD, 17, 057]. They are consistent with three different types of ovals, and they all share the common feature that the origin of the vertical tangent to the minor axis of the oval is located one foot below the impost line (Figure 6).

The main ovals' geometric data are shown in Table 1, where:

- e1 describes the clear span;
- e2 describes the rise;
- a1 describes the distance between centers of the minor axis;
- a2 describes the distance between the center of the minor arc and the minor axis;
- d1 describes the ratio between the length of the semi-major axis and the vertical distance from the semi-minor axis to the springline (feet);
- d2 describes the ratio between the length of the minor axis and the distance from the center of the major arc to the point of tangency between the major arc and the minor axis;
- d3 describes the ratio between the length of the semi-major axis and the radius of the oval's minor arc;

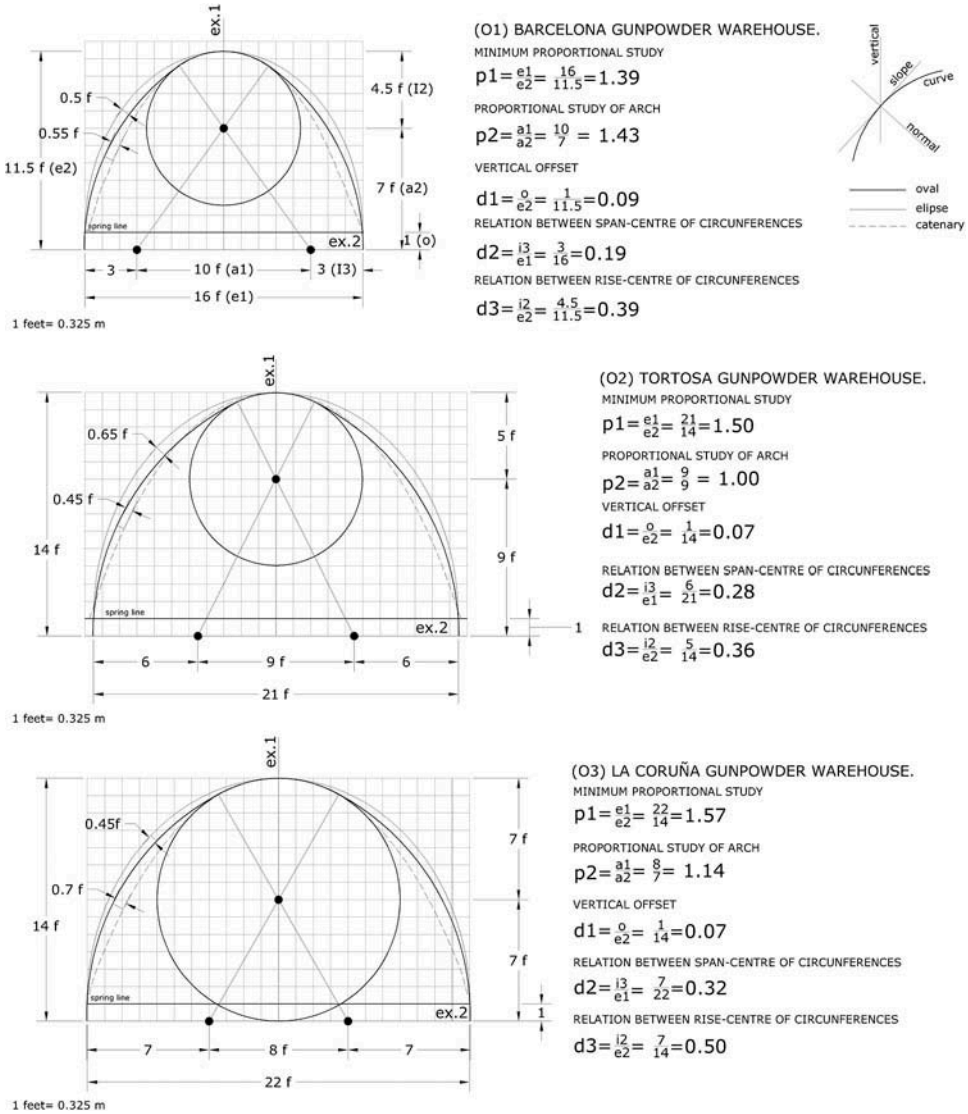


Figure 6. Geometrical analysis of the vaults of the projects for the gunpowder magazines.

Table 1. Geometrical characteristics of the ovals for the gunpowder magazine projects

Ovals' geometric data												
Oval	**e1** (m)	**e2** (m)	**e1/e2** (m)	**a1** (m)	**a2** (m)	**a1/a2** (m)	**d1** (m)	**d1/e2** (m)	**d2** (m)	**d2/e1** (m)	**d3** (m)	**d3/e2** (m)
O1	16	11.5	1.39	10	7	1.43	1	0.09	3	0.19	4.5	0.39
O2	21	14	1.50	9	9	1.00	1	0.07	6	0.29	5	0.36
O3	22	14	1.57	8	7	1.14	1	0.07	7	0.32	7	0.50

- p1 describes the ratio between the semi-major axis and the minor axis; and
- p2 describes the ratio between the center to center distance on the minor axis and the distance from the center of the semi-major axis to the minor axis.

The ovals that are used in the layout of the gunpowder magazines are thus used as a reference for purposes of comparison with the cellar's layout. The layout of [O1, O2, O3] is based on a ratio between d3 and e2 of [0.39; 0.36; 0.50], respectively (Table 1).

In addition, the layout of each oval is compared with a catenary that has the same rise and span, which is drawn using Innersoft software. According to the results, the inner surface that is defined between the corresponding geometric shape and the springline is different (1.33 m^2 vs. 0.98 m^2). Furthermore, the ratio between the maximum distance between the geometric shapes and the arch's span ranges from 2.1–3.44%. From these data, we can conclude

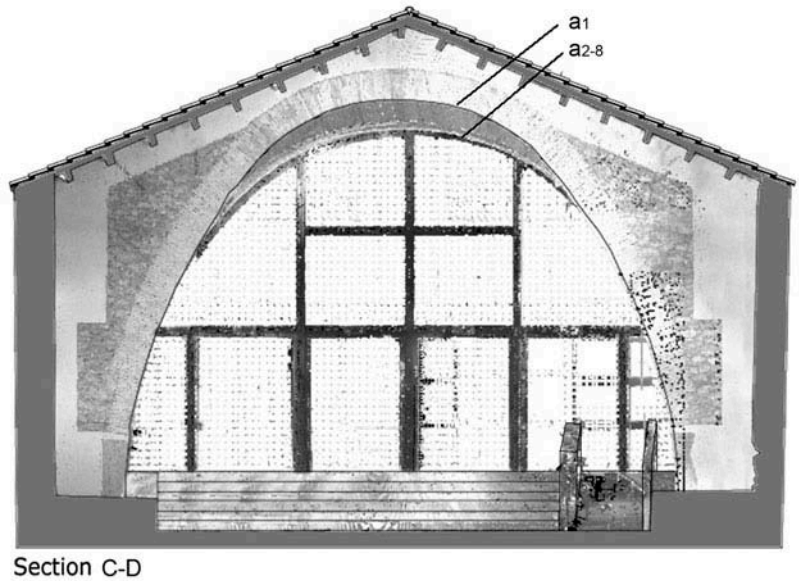


Figure 7. Transversal section of the O'Connor cellar in Benicarló (1757).

that the approximation that was made by the engineers by drawing ovals in the three projects that are considered in this study is sufficiently precise for the drawing scale that was used, between E: 1:90 and E: 1:70. Finally, the curves are compared with an ellipse with the same rise and span. The obtained shape is clearly not coincident with the curves of the projects.

5. Evidence in civil constructions

The O'Connor family Benicarló's building was built in 1757. The Carlón wine cellar (Constante 2012), has a rectangular floor plan, and its inside measures are 12.42 m in width and 43.01 m in length (Figures 7 and 8). In the nave, there are eight diaphragm arches, each having a single two-piece offset-jointed ring and an average depth of 0.60 m. The arches are made of solid ceramic bricks (measuring 0.37 x 0.18 x 0.04 m), and they rest on a limestone base that was brought from Santa Magdalena de Pulpis. The top of this base determines the springline of the ceramic arch. The arch's abutment and the outside walls are made of ordinary uneven masonry.

The geometric study of the cellar arches is based on the topographical survey that was conducted with a laser scanner by J. Lluis, A. Costa, and S. Coll (2014–15) (Figure 7). The formal characteristics of the arches are different (Table 2): arch a_1 has a clear span of $e1_{a1}$ = 9.65 m and a rise of $e2_{a1}$ = 5.82 m, whereas the other seven arches can be grouped together. Their span is within a range of $e1_{a(2-8)}$ = [9.76–9.69 m], which is similar to arch a_1, but their rise significantly differs from the first arch, within a range of $e2_{a(2-8)}$ = [5.46–5.45 m]. All of the arches share a special feature: they do not have a vertical tangent on the stone base. The angle of incidence (α) of these arches with respect to the vertical for both sides, left (α_a) and right (α_b), and have following values: $\alpha_{a.a1}$ = 13.94º and $\alpha_{b.a1}$ = 8.97º in arch a_1 and $\alpha_{a.a(2-8)}$ = [4.49º- 1.80º] and $\alpha_{b.a(2-8)}$ = [7.38º-2.58º] in arches $a_{(2-8)}$ (Figure 8). By statistically analysing the parameters, for arches $a_{(2-8)}$ the average span calculated is $e1_{a(2-8)}$ = 9.72 m, and the average rise is $e2_{a(2-8)}$ = 5.46 m.

Table 2. Geometrical characteristics of the arches

Arches' geometric data (m)				
Arch	**e1**	**e2**	**αa**	**Ab**
a1	9.65	5.82	13.94	8.97
a2	9.74	5.46	1.80	5.90
a3	9.69	5.46	2.09	5.59
a4	9.72	5.45	2.87	5.88
a5	9.73	5.46	1.54	7.38
a6	9.76	5.46	3.65	4.72
a7	9.75	5.46	3.63	5.94
a8	9.71	5.45	4.49	2.58

It seems that the measurement units that were used for the construction of the cellar were the toise (194.90 cm) and the toise foot (32.48 cm) (Lluis I Ginovart 2013). Arches $a_{(1-8)}$ have an average span of 29.92 toise ft (9.71 m), with an error of 0.02 m for 30 ft (five toises, 9.74 m). The rise of arch a_1 is 17.92 ft (5.82 m), i.e., there is an error of 0.02 m in 18 ft (5.84 m), which are three toises. Arches $a_{(2-8)}$ have a rise of 16.80 toise ft (5.46 m), i.e., there is an error of 0.06 m in 17 ft (5.52 m). The arches are 0.60 m in width (1 + 10/12 feet). Regarding the outside measurements,

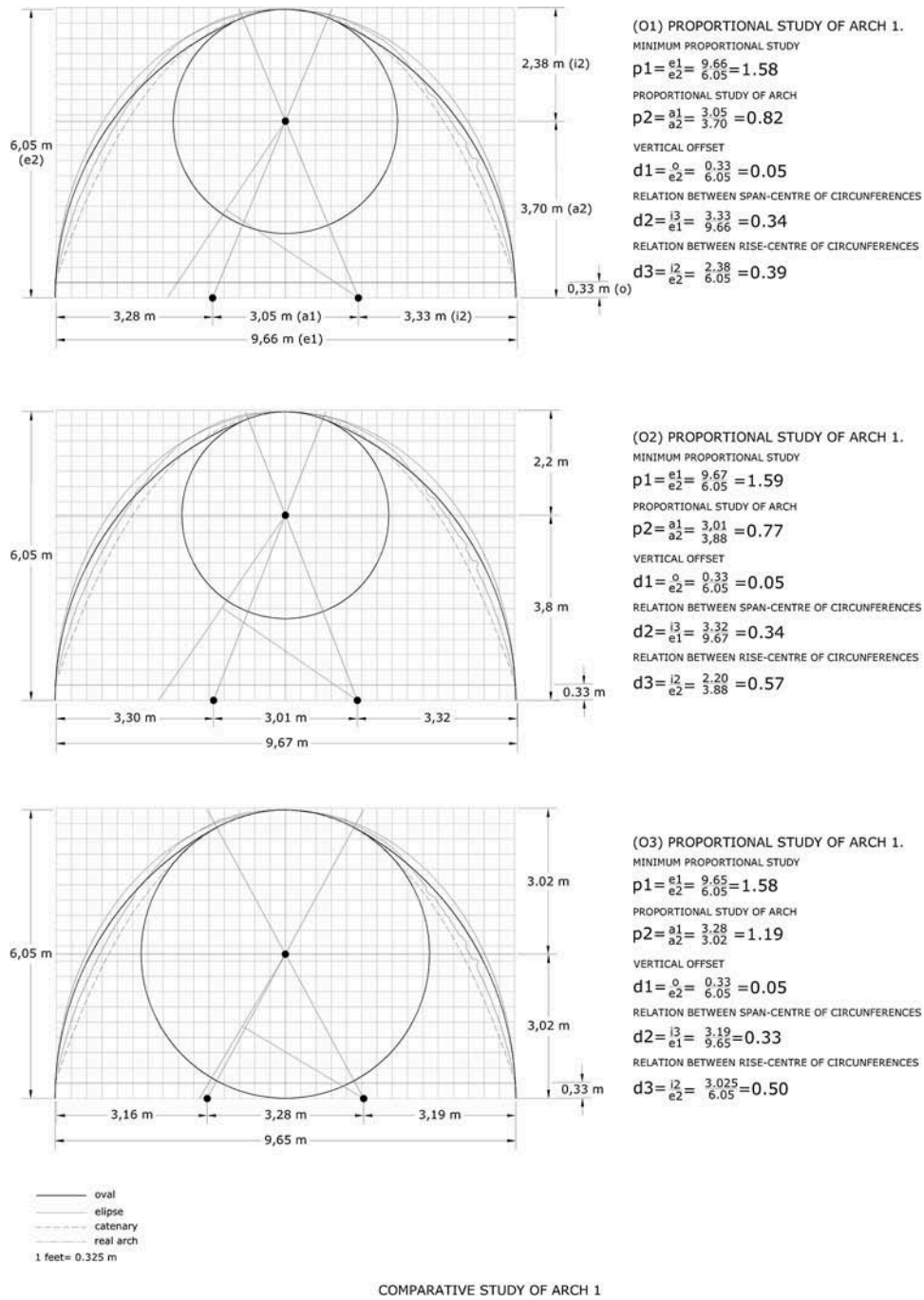


Figure 8. Geometrical analysis of the arch 1 in the O'Connor cellar (1757).

the nave is 41.50 ft (13.48 m) wide and 92.36 feet (30 m) long, and the arches' abutments are structures measuring 5 + 9/12 ft. The inside length of the cellar is 43.01 m, i.e., 132 + 5/12 toise ft. The enclosure wall on the façade is 2 ft thick; thus, the span-to-arch ratio is 5.75/30 ft (Figure 9).

A metrological analysis of the arches in Benicarló's cellar reveals that the eight arches show the same metric relations, i.e., a five toise span and a three toise rise. The dimensions of the catenary arch a_1 are 30 x 18 feet (exactly 5 x 3 toises). The dimensions of the elliptical or oval-shaped arches $a_{(2-8)}$ are 30 x 17 feet. If we follow the hypothesis that the minor axis (either the ellipse minor axis or the oval minor axis) is one foot below the impost, then the geometric relationship of arches $a_{(2-8)}$ is also 30 x 18 toise ft.

A statistical analysis is then performed on each of the eight arches $a_{(1-8)}$ to determine the difference between the shape of the arches built and the shapes of reference: ellipse (E), catenary (C), and ovals

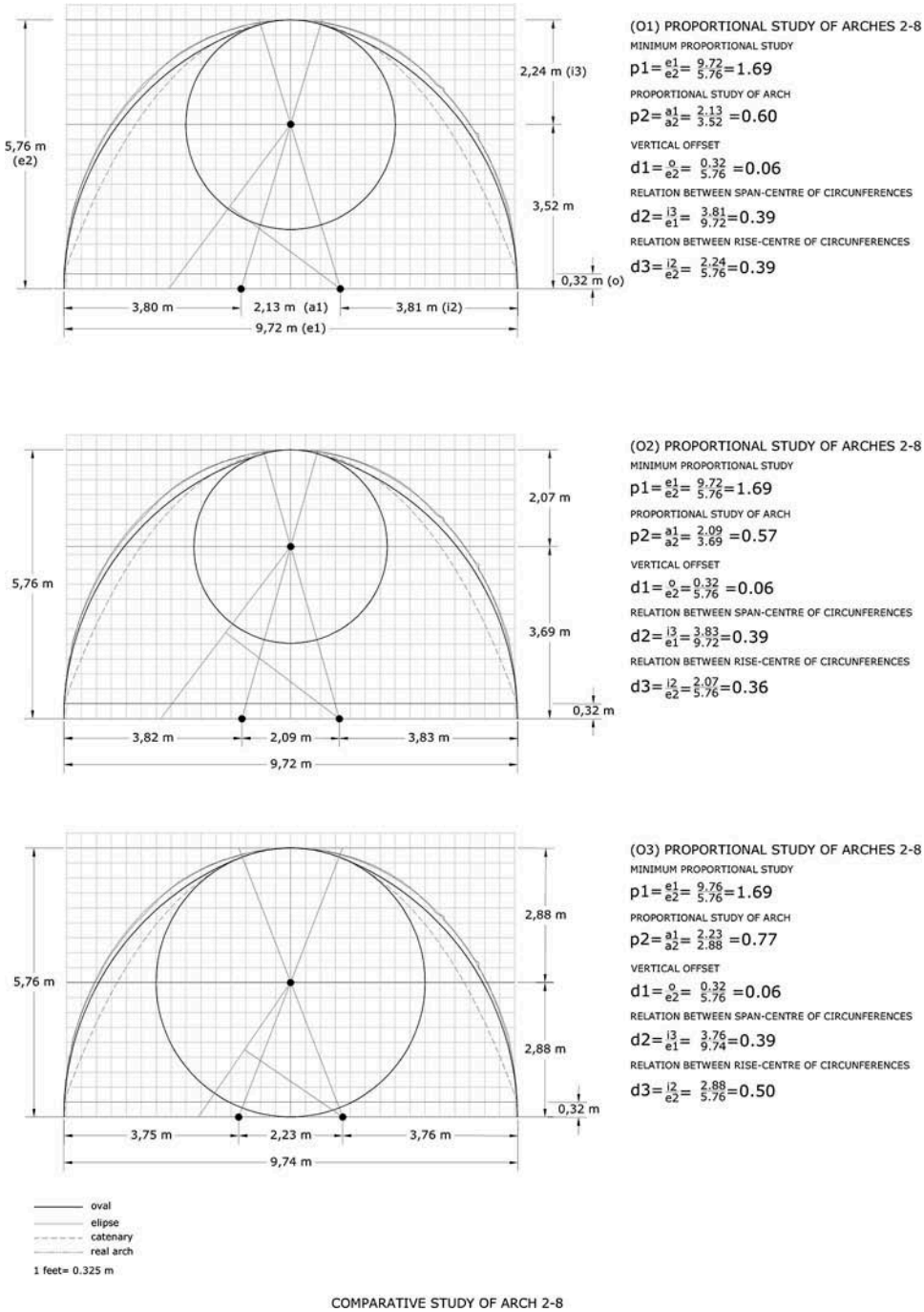


Figure 9. Geometrical analysis of the arches 2–8 in the O'Connor cellar (1757).

[O1, O2, O3]. The following values are calculated for 29 points on each cellar arch.

(a) The average and maximum deviation of these 29 points (Table 3); and
(b) the angle of incidence on the springline (Table 4).

The mean deviation has a spread of only 0.03 m, which is approximately 0.31% of the arch's span, which makes it difficult to conclude whether it is a catenary or an oval. The determining feature is that arch a_1 has an angle of incidence on the springline [$\alpha_{a.a1}$ = 13.94º, $\alpha_{b.a1}$ = 8.97º]. Because the catenary's angle of incidence is 19.38º, the geometric shape that most closely resembles the arch is the catenary.

Conversely, the statistical analysis of the remaining 7 arches $a_{(2-8)}$ shows that the geometric shape that they most resemble is the ellipse, with an average deviation that ranges between [0.001–0.015 m]. The range for oval-shaped arches is [0.006–0.186 m] (*arco apaynelado* or *arco*

Table 3. Mean deviations of the ellipse, the catenary, and the ovals with respect to the arches

Arches' deviations (m)						
Arch	**Deviation**	**ai-E**	**ai-C**	**ai-O1**	**ai-O2**	**ai-O3**
a1	**average**	0.119	0.115	0.100	0.107	0.086
	max.	0.290	0.211	0.176	0.185	0.186
a2	**average**	0.015	0.228	0.105	0.111	0.087
	max.	0.041	0.425	0.188	0.199	0.171
a3	**average**	0.021	0.234	0.124	0.128	0.104
	max.	0.079	0.449	0.241	0.246	0.220
a4	**average**	0.020	0.239	0.131	0.135	0.114
	maxim	0.044	0.467	0.212	0.219	0.192
a5	**average**	0.043	0.274	0.158	0.159	0.133
	max.	0.094	0.534	0.261	0.262	0.240
a6	**average**	0.035	0.276	0.138	0.141	0.123
	maxim	0.089	0.583	0.254	0.256	0.235
a7	**average**	0.034	0.265	0.144	0.152	0.132
	max.	0.073	0.505	0.237	0.247	0.222
a8	**average**	0.031	0.264	0.143	0.145	0.127
	max.	0.058	0.476	0.225	0.228	0.203

carpanel according to Tosca, i.e., three-centered arch or basket-handle arch; or *anse de panier* according to Bélidor), so the curves that were used by the Bourbon engineers to lay out the projects for the gunpowder magazines are very similar to the cellar's arches. Nevertheless, the angle of incidence on the springline tends not to have a vertical tangent, which is a fundamental feature of both the ellipse and the oval in the arches that are considered here. In the springline of these arches, the angle of incidence ranges between [$\alpha_{a\ .a5} = 1.54^{o}$, $\alpha_{b\ .a5} = 7.38^{o}$] (Figures 9, 10).

Thus, arch a_1 resembles a catenary arch, whereas the other seven arches a_{2-8} tend to be ellipses. These seven arches do not have a vertical tangent on the springline because their horizontal axis has been moved one foot below the arch's springline. As defined by Frézier (1738), the shape of the catenary has the following essential property: the vertical line, which is tangent to the curve at the springline, does not form a right angle with the horizontal plane. Therefore, geometrically, the catenary can be understood as any curve that does not have a vertical tangent at its springline. This is what happens in the springline of St. Paul's dome in London (Heyman 1988), which was designed by Christopher Wren in collaboration with Robert Hooke (Addis 2013). Otherwise, it should be noted that from a mechanical perspective, catenary arches are an optimal solution to build masonry arches, because the material has very low tensile strength.

Finally, from the construction point of view, the catenary shape can be approximated using other geometric forms such as ovals or ellipses, under the condition that there is not a vertical tangent at the springline. The catenary shape forms a barycentric axis, which minimizes the tensions on a linear element that is subject to only vertical loads. In the arch, the inverted catenary shape prevents the appearance of stresses other than compression stresses.

Thus, there are two hypotheses regarding the construction of the wine cellar. The first one is that the construction work was started from the inside toward the facade, and arches $a_{(2-8)}$ were constructed before the catenary arch a_1. The second hypothesis is that the construction work began with arch a_1. According to the second hypothesis, there is also a difference between both types of arches: on the first brick courses from the springline of arch a_1 (the first nine courses on one side and the first 17 courses on the other side), the ring is 0.36 m wide. On the remaining seven arches, the ring is 0.60 m wide (just like the arch's depth). It is clear that less ceramic material is necessary for the construction of arch a_1 than for the other seven elliptical arches $a_{(2-8)}$ (Figure 10).

6. Conclusions

The assessments of the gunpowder warehouses by Miguel Marín for Barcelona (1731) and Tortosa (1733) and by Juan de la Feriére y Valentín in A Coruña (1736) are only a mere 4.05% of the projects that were analysed. However, they prove the intention to lay out the vault as a catenary. These authors knew that in a catenary the tensiles in the shape of a hanging chain have the same compression values in the inverted geometrical figure. These engineers had a vast knowledge of the mechanical principles of the modern theory of masonry. From a scientific perspective, catenary vaults are the most interesting because they introduce the principles that were established by Hooke (1676). Both the arches of gunpowder magazines and the arches $a_{(2-8)}$ of Benicarló were laid out using the geometrical construction

Table 4. Angles on the impost line of arches, ellipse, catenary, and ovals

Arches' geometric data							
	Angle of arches a(1–8) (α°)		**Angle of reference shapes** (α°)				
Arch	**αa**	**αb**	**Catenary**	**Ellipse**	**Oval O1**	**Oval O2**	**Oval O3**
a1	13.94	8.97	19.38	3.36	3.74	4.11	3.75
a2	1.8	5.9	19.57	2.52	3.46	4.81	5.77
a3	2.09	5.59	19.87	3.44	3.39	5.11	4.84
a4	2.87	5.88	19.61	2.93	5.39	3.29	3.18
a5	1.54	7.38	20.45	2.92	3.16	4.12	5.28
a6	3.65	4.72	21.05	4.27	4.51	5.69	6.21
a7	3.63	5.94	19.85	3.74	3.59	2.69	5.61
a8	4.49	2.58	20.11	2.69	3.35	3.12	3.82

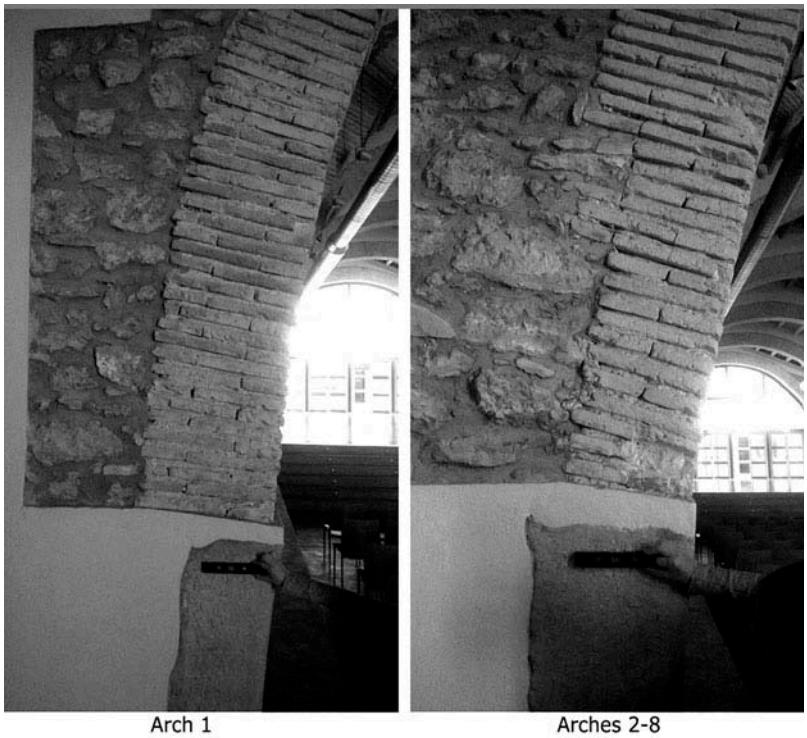


Figure 10. Springing of arches no. 1 and no. 2 in the O'Connor cellar.

of an oval. Otherwise, the location of the horizontal axis of the ovals under the spring line reveals the application of one of the characteristics of the catenary. This means that the vertical line, which is tangent to the curve in the springing, does not form a right angle with the horizontal, so they used the chain theory in the layout of the projects.

Formally, if the distance between the axes and the springline of the arch is small, then the angle of incidence has a minimum influence on the thrust and the line of pressure. Otherwise, the location of the axis under the springline reveals the intention to minimize stresses at this point and in the neighboring areas, although the final mechanical influence is small.

While there is no evidence of the construction of gunpowder warehouses, it is possible to confirm the use of catenary arches in the construction of the Carlón cellars of the O'Connor in Benicarló (1757). There are significant differences between the measures of the arches of the gunpowder magazines (maximum span: 22 ft; maximum rise: 14 ft) and arches of the Benicarló cellar (span: 30 ft; rise: between 17 and 18 "toise" ft, until the springline). In addition, the span-to-rise ratio of the oval arches in the gunpowder magazine studies is [1.39–1.57], whereas in Benicarló, this ratio is [1.67–1.76].

It can be concluded that arch a_1 is a catenary arch, whereas arches $a_{(2-8)}$ (Figure 2) tend to be elliptical. Arches $a_{(2-8)}$ show the special feature that their (x) axis is located below the springline; therefore, the tangent of the curve on the springline does not form a right angle with the horizontal. This is a feature of the definition of the catenary.

The theory of the equilibrium curve, which is followed by most British engineers, became known to Bourbon military engineers through the Academy of Mathematics in the eighteenth century and was used by some immigrants of English origin, such as the O'Connor family, a century before the modernist architecture of Antonio Gaudí.

ORCID

Josep Lluis I Ginovart http://orcid.org/0000-0001-5957-762X
Sergio Coll-Pla http://orcid.org/0000-0002-4718-5810
Agustí Costa-Jover http://orcid.org/0000-0002-6194-3243
Mónica López Piquer http://orcid.org/0000-0002-3945-5462

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