Graphic techniques for the constructive identification of masonry vaults

J. Lluís, A. Costa, S. Coll & J. M. Puche
Universitat Rovira I Virgili, Spain

ABSTRACT: Architecture use to be represented by the “iconography” of the floor plant, incorporating the projections of the ceiling by means of dashed lines. Sometimes, the expressivity of architecture is produced by the vaulted covering. The representation of the plane of the roof is a problem because the points are inaccessible for direct measuring. It has been done three campaigns to draw the gothic cathedral of Tortosa (Spain), using the following techniques: tachymetry (2000), photogrammetry and tachymetry (2012) and laser scanning (2013). The paper presents the methodology and results of the 2012 campaign, focused on the assessment of the heptagonal apse. By means of technological resources available to any professional, it has been obtained an accurate “iconography” of the vaulting. Geometrical anomalies of the masonry vaults are defined and analyzed by means of computer processing of the data using a 3D model, with an accuracy impossible to reach before. These results reveal new information about the construction process of the gothic apse.

1 INTRODUCTION

The detailed study of masonry vault has always set the problem of accessibility. Direct techniques of measuring need a complex and expensive scaffolding system. Thus, a cheaper and faster technique based on photogrammetry is proposed to assess the vaults of the apse of Tortosa Cathedral, built between 1383 and 1441. The Cathedral heading is composed by 27 vaults: 9 in the presbytery; 9 in the ambulatory and 9 in the edge chapels.

Figure 1. Tortosa Cathedral apsis (1383-1441).
Tortosa Cathedral has an apse with double ambulatory (1383-1441). Constructive chronology has been determined through the “Llibres d’obra” (Ll.o., cathedral construction accounts) (Almuni, 2007), preserved in the Chapter Archive (ACTo.). At the beginning of the construction, the Master imposes the unit of measure for the new cathedral (1347), in this case the “cana”; “Item fiu lo maestre de/la obra a-n Antoni ferrer una cana de ferre per prendre mesures de l’obra costa….V.s” (Ll.o.2 1345-1347, f.36 v) (Almuni, 1991). The “cana” has eight palms, and a palm has twelve fingers. The “cana” of Tortosa is defined by the Book IX, Rubric 15.5 of Consuetudines Dertosae (1272) (ASCTE, cod.53, fol.256r). The “cana” is also defined in the copy of 1346, Llibre de les Costums Generals feutes de la insigne ciutat de Tortosa (FBMPM, fol.100r). The construction of the new gothic cathedral started along 1346. Comparing documents about the “cana” of Tortosa and the “cana” of Barcelona (24-VII-1593), it is determined that the “cana” of Tortosa used in the Cathedral, has 1.858cm and a palm of 23.23cm¹.

1.1 The tradition of geometric section in architecture

The geometric conditions which inspire the theoretical gothic project determine the reference parameters of the section. The study of the development of medieval masonry section is complex, as medieval builders were held to professional secret of geometria fabrorum (Reched, 1980). The Saint Michel of Strasbourg Status (1563) banned showing the development of the section from the main points of floor plant, using a carpenter’s square. (Strasbourg, 1563, art.13) (Mathionère, 2003). The construction of cathedral of Milano (1386) and the later discussions (1392) and (1401) allow an epistemological approach to determine the methodology of these geometric and arithmetic developments.

The Master builders of Milan discussed about the use of ad triangulum, ad cuadratum French model or other lower proportion defended by Milan masters (Panofsky, 1945; Frankl, 1945; Ackerman, 1949; Beaujouan, 1963). C. Cesarino Vitruvian edition illustrated the section of Cathedral of Milan (Vitruvio, 1521). Cesarino explained the method to know the main vault height from its width and also the width of edge chapels.

The proportional gothic theory, derived from this medieval tradition, has direct application: The ad cuadratum section has a proportion of 1/1, while the ad triangulum section is based in the relation of width and side of equilateral triangle. During Gothic period, the width of equilateral triangle was calculated approaching square roots of number three. The immeasurability of the measure causes that Gerverto de Aurillac (997-999) proposed the 7/6 proportion as a solution (Buvnov, 1899). Gabriele Stormalocho (1391) use the 8/7 proportion in Milan cathedral (Beaujouan, 1963). Studies in Gothic buildings indicate the application of ad cuadratum and ad triangulum proportions in the development of the section (Lund, 1921). Other smaller sections, Pitagoric sections or Neoplatonic sections with harmonic modulation, has been also suggested (Hiscock, 2000). This is the reference framework of theoretical section in Gothic buildings, where exist a proportional relation between the points in the floor plan and their respective section points. With this information it is determined the theoretical section of reference in Tortosa, in order to find punctual differences in masonry vaults.

1.2 The model for the apse of Tortosa Cathedral (1383-1441)

The assessment of the vaults is referred to their theoretical section, according to the floor plan measures: 9/5 in the radial chapels with 10.45 m in height; 9/6 in the ambulatory with a height of 17.73 m; and 9/6 in the presbytery with 23.23 m in height. There are documental sources where it is established the sequence of vault construction, as a trapezoidal sector in the ambulatory, located in the Chapel of San Vicente, which area measures 20.03m². The scaffold is built in 24 July of 1433, the formwork is installed in 31 July of 1433, vault is built between 1 and 5 of August and the scaffold is removed the 27 August of 1433. The construction was finished in 35 days.
Between 2 and 13 of September it was build the filling of this vault, the inner wall, and the external pavement. This pavement was built in two different layers, finishing the masonry work with this action. Thus, a vault of the ambulatory was covered in only 8 weeks (Almuni, 2007). The masonry arch, after removing the formwork, starts pushing against the buttresses, which will yield slightly. The width of the arch increases to accommodate this movement (Huerta, 2004).

This is not the only source of geometrical deviations in the theoretical section. Deformations can be also produced by implementation and application techniques using different auxiliary resources (a). It depends of geometry, height of the framework and tensions of the vault. It also depends of mortar joints. Other reason is the deformation of formwork by recurrent use.

Moreover, another reason can be the settlement of vaults after removing the formwork (b). A movement and settlement is produced in the buttresses after unshoring. Sometimes the central keystone dawns as consequence of joint consistency and sternotomy of the vault. Over time, masonry settlements can also take place, affecting buttressing contour.

The height of vault framework is not constant in radial chapels, as observed through the evidences in the masonry. The variation is between 5.94 and 8.29m. Otherwise, the imprint line of ambulatory is constant over 13.12m. The presbytery is the same case, where eleven different masonry carvings can be perfectly differentiated at a height of 17.09m.

2 METHODOLOGY

The first computerized topography of Tortosa cathedral was made between 1995 and 2000, for the Santa Maria Dertosae Master Plan (Lluis & Llorca, 2000). The field of work was performed by direct measuring and rectified through polygonal points fixed with a total station. The method has a good accuracy in the floor plant, but it is not optimal in the ceiling.

To draw the vaults it is performed a 3D model through photogrammetric techniques. The data capture takes place during summer of 2012, with the combined data of a Total Station Topcon Imaging Station 203, 0.2mm/1mm+- 5mm² of accuracy, and exhaustive photographing with a NIKON D7000+Tokina 12-24, using the extreme focus zoom is used (12mm and 24mm).

The cathedral gothic apse is surveyed with 221 points through five different positions, four points in the ambulatory and one over the main axis of the central nave. The five captures have to be placed in the same Cartesian coordinate system. Thus, singular points of keystones of vaults are used as reference points. The reference points are obtained with an error lower than 10mm.

The 3D model is generated by surfaces. It is made from pairs of photograph oriented by topographic points (control points). The surfaces are generated with software Image Master (Topcon), obtaining a TIN mesh (triangulated irregular network). The TIN mesh incorporates the texture of the real masonry with an average triangle measure of 30mm in optimal conditions. The result is a 3D model with a texture incorporated of parameters studied. Orthophotographs in metric scale, contour models and masonry vaults sections are processed through the 3D virtual...
model. The result must be studied from statistics. So is necessary the selection data and parameterization of results (Rosin, 1996; Rosin & Pitteway, 2001).

Measures of vaults in a (x, y, z) coordinate system is directly taken from the 3D object (Fig. 5). The method allows obtaining a bidimensional planimetric representation. This system represents the vaults as a topographic plan with orthogonal projection and contours (Bechman, 1981) (Fig. 6).

The assessment is carried sectioning the severies according to axial coordinates. A nomenclature is also established in order to identify characteristic points of the geometry (Fig. 4). The chapels are identified with a C and a numeration in clockwise direction from 1 to 9, starting from San Pedro chapel. The vaults of the ambulatory (G) is numbered with the same criteria.

After, the characteristic points of vault geometry is identified. Radial section is referred as S.I and transversal section as S. II. Four characteristic points are identified for each one:
- Keystone height (Ha).
- Vault height in assembly of key (Hb).
- Vault height in assembly of (Hc).
- Perimeter wall (in S.I).
- Bondstone arch assembly with next vault (in S.II).
- Vault height in assembly with (Hd).
- The arch of aperture of chapel to ambulatory (in S. I).
- The bondstone arch in contact with previous vault (in S.II).

When the characteristic points are determined, the numerical values are used for the comparative study of geometrical deviations of the vaults. Apparently there are not significant differences between different vaults of the same typology, but its topography is not uniform and the movements can be quantified.

Figure 5. Planimetric projection of vaults from 3D Model. Heat of Tortosa Cathedral (2012).

Figure 6. Contourn of vaults. Tortosa Cathedral (2012).
3 RESULTS

The height of vault keystones in chapels presents a variation of 39.1 cm. The highest keystone is in vault C2 (10.285m) and the lowest is in vault C8 (9.894m) (Fig. 7). A progressive loss of height is observed from vault C1 to C9, being the first three vaults the highest. C2 and C3 have similar height, with a difference of about 6 cm, while the last four are lower. C7 is the highest with a height of 10.068m. The height of severies in the key (Hb) (Fig. 7) has the same explained pattern in the keystone with little variations.

![Figure 7. (Ha) vault key height (Hb) severie height into key.](image)

The comparison between Hc and Hd in S.I (Fig. 8) reveals that HC always is always higher than Hd. The difference of height is 25.2 cm in the first (from C4, 10.589 to C3, 10.337 cm) and 17.8 cm in the second (from C8, 10.307 to C3, 10.129 cm). The three first vaults are higher than the rest. C2 is the highest with 10.441m in height in Hc and 10.277 in Hd. C3 is the lowest.

In the case of S.II (Fig. 8) there are a higher difference of values, not being true Hc>Hd. In the other hand, the three first chapels have lowest height value. The height is between 17.3 cm to Hc (C9 10.445 m and C4 10.272) and 18.6 cm to Hd (C8 10.414 m and C1 10.228).

![Figure 8. Heights in chapels.](image)

In the case of the ambulatory, the height of the keystone (Ha) (Fig. 9) presents a different pattern than in chapels. Its height is ranged between 23 cm (G1, 16.152 and G3, 15.919). Extreme vaults are the highest, and height tends to decrease, rising again in the center. This effect can be observed in Figure 9. Again, the high of severies (Hb) (Fig. 9) has the same pattern.

The relation of heights in the vaults of ambulatory is different than in chapels. Hc is lower than Hd in S.I. (Fig. 9), except in G8. The deviation in Hc is 12.4 cm (G1, 16.247 to G8, 16.371) and the deviation in Hd is 29 cm (G1, 16.309 to G4, 16.28).

The high ranges between 11.9 cm in Hc (G6, 16.44 to G9, 16.321 m) and 13.5 in Hd (G1, 16.31 to G5, 16.445) in the case of S. II (Fig. 9). The three central vaults are higher (G4, G5 and G6), with little differences in height.
4 DISCUSSION

The comparative analysis between the sources about the construction of the masonry gothic apse and the results of the assessment through the 3D model allows a bijective action. Moreover, the studies of reference of gothic vaults will frame the typology of Tortosa cathedral absis with its double ambulatory (Willis, 1842; Viollet le Duc, 1854-1868; Viollet le Duc, 1996; Ungewitter, 1890-1892; Babcock, 1893; Drum, 1901). In typological terms, the edge chapels has visual connection between them, as the traditional separation wall is replaced by an arched system. The elimination of this wall was tested in Santa Maria de la Aurora of Manresa (1328) with an ad triangulum section of 8/7. The wall was completely removed in Tortosa 1377, using a lower section of 9/5. The same structural question will appear some years later into Millan discussion (1392) (Yarza, 1982; Valentini, 1990).

4.1 The vaults of the edge chapels (1383-1424)
Four Magisters successively ruled the construction of the edge chapels. They conducted different adjustments in the moldings and the cutting of the masonry. Three building moments can be identified through the sources in the construction of chapels. These moments can be understood as an empiric experiment. The first phase (1377-1383) involves the first test of the model with the construction of the chapels of San Pedro, San Pablo, and San Vicente (C1, C2 and C3). Between 1387 and 1397 the following two chapels are built, it meant the consolidation and adjustment of the model. Finally, the third constructive phase takes place between 1412 and 1424, when the building system is systematized and the four last chapels are built.

The results of the geometrical assessment agree with the different constructive phases identified through the sources. The keystone of the first three vaults is higher than the following, and also the heights have a different pattern (Fig. 11). C4 breaks with the previous pattern, and there is a successive stabilization of the pattern, with the keystone at a lower height and similar relationships between the other points. The pattern changes significantly once again in the last two chapels (C8 and C9), which can mean another change in the mastership which was not registered in the LI.o.

4.2 The vaults of the ambulatory (1432-1434)

The ambulatory vaults were built symmetrically from central axis between 1432 and 1434. The edge chapels were built correspondingly. In this case, the construction phases follow a different pattern than in chapels. Thus, the ambulatory is covered symmetrically from the gospel and epistle. The square vaults were covered on first place, and after that the trapezoidal vaults were covered subsequently. The Figure 12 shows a pattern in the variation of heights in the vaults that match with the construction process known through the sources.

5 CONCLUSIONS

The results of the assessment show the potential of a simple but accurate data collection in order to know and evaluate the geometrical movements of arches and vaults. This study reveals the
anomalies of the covering regarding to the theoretical section. The technique used is currently available to any professional, and almost don’t need specific technology. It enables the orthogonal representation of the plane of the roof, but is ineffective in the assessment of the vertical plane, as the architectonical configuration causes too many occlusions.

The study allows the parameterization of the geometry of vaults. The guidelines of vaults have been assessed, rising to light some structural and constructive details about the radial chapels and ambulatory. The layout of the ribbed vaults was well known, according to some treatises (Viollet-le-Duc, 1854-1868; Viollet-le-Duc, 1996) as well as the higher location of the keystone of the vault (Lluis, 2009).

The geometric variations related to the theoretical section are hard to determine from obtained data. It is impossible to distinguish between the movements during the construction process, during the formwork of vaults (typology a) and because of the decentering (typology b). The geometry after this process is absolutely determined through the methodology, although it is very different from its initial geometry (Barlow, 1846; Jenkin, 1876).

The methodology reveals new information compatible with the direct sources of masonry books (Ll.o.). Thus, the several Masters can be recognized through the geometric variations between vaults. So, it is can be a powerful tool in the assessment of historical masonry vaults, completing the historical data of a construction.

REFERENCES


